Introduction to Operations Research

An Example of OR application

- You are in-charge of tool room. There is a single jig boring machine. Five jobs are to be done in that machine. Machining time for the jobs are 50, 70,120,180 and 60 min. How should you schedule the jobs to minimise time loss?

Introduction

- **Operations Research** is a branch of mathematics which is concerned with the application of scientific methods and techniques to decision making problems
- OR is used for problems ranging from management science to design optimisation

Agenda

- In this course we shall try to
  - Briefly touch on different techniques of OR
  - Possible application areas
  - Understand the Tools easily available for solving routine problems
- Objective: to be able to make better decisions

Methods of Operations Research

Mathematical programming techniques	**Stochastic process techniques**	**Statistical methods**
• Calculus methods • Calculus of variation • Nonliner programming • Geometric programming • Liner programming • Dynamic programming • Integer programming • Stochastic programming • PERT & CPM • Game theory	• Statistical decision theory • Marov process • Queueing theory • Renewal theory • Simulation methods • Reliability theory	• Regression analysis • Cluster analysis • Patteren recognition • Design of experiment • Deterministic analysis

Classification of OR problems

- Answer type:
  - Descriptive
  - Maximization/ Minimization
- Boundary of problem
  - Bounded
  - Unbounded
- Solvability
  - Solvable under all circumstances(Polynomial problems)
  - Solvable for small problems but partically unsolvable for big problem(Combinotorial problem)
  - Unsolvable
  - Unknown, our knowledge is not sufficient to solve the problem.

Answer to the Scheduling problem

Let the machining time for ith job be T_i. Then total waiting time is

$w = T_1 + (T_1 + T_2) + (T_1 + T_2 + T_3) + \ldots$

$= nT_1 + (n-1)T_2 + \ldots$

It can be seen minimal waiting time is attainable when $T_1 < T_2 < T_3 \ldots$

In fact this rule for scheduling is know as SPT (Shortest processing time)

Steps for solving OR problem

- Formulate the problem to be solved
- Build a mathematical model
- Select appropriate tool necessery to solve the problem
- If necessery, simplify by introducing appropriate assumptions
- Perform the analysis
- Implement findings

Calculus method for solving

- Finding Maxima and minima of a curve is one of the fundamental tool of OR application
- Example: For a material each delivery costs Rs.300/ -. Averave requirement of the material is 100 per day. Cost of the material is Rs. 5/ - per pc. If annual interest charge on capital blocked in inventory is 15% what is the most economical ordering quantity?

Economic Order Quantity

Let, D - demand per day

Q - is order quantity

i - interest charge

C - ordering cost

Average cost per day (W) =

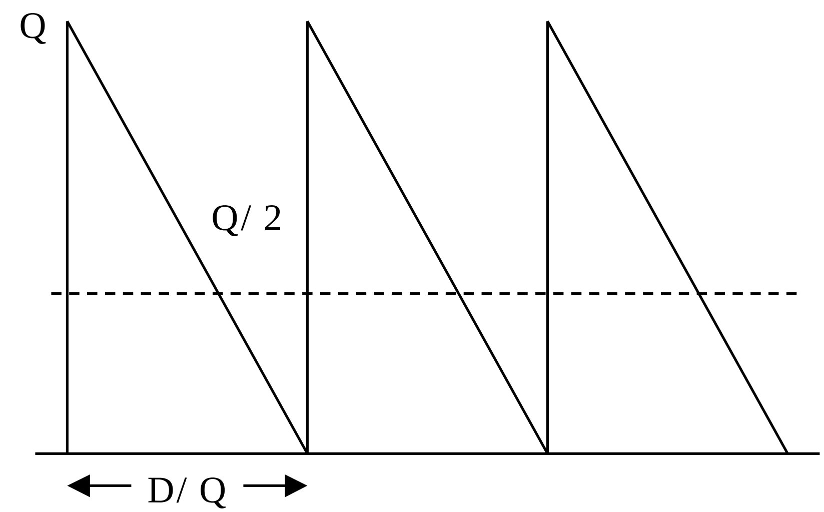
Ordering cost = CD/ Q

Purchase cost = D

Holding cost = iQ/ 2

W= CD/ Q+D+iQ/ 2

- $\left. dW/dQ \right|_{Q=Q^*} = 0$ at optimal Q



$$\therefore\ i/2 - CD/Q^{*2} = 0$$

$$Q^* = \sqrt{\frac{2CD}{i}}$$

Ordering policy

- Hence Economic order quantity =

  $(2 * 300*100*5/ (.15/ 365))^{1/2}$

  = 2708
- ie: We should order once in a month for this material

Liner Programming

- *Liner Programming (LP) is an optimization method where both objective function and the constraints are liner functions of decision variables.*
- Major application of LP is in
  - Product mix optimization
  - Cutting of bars
  - Resource allocation

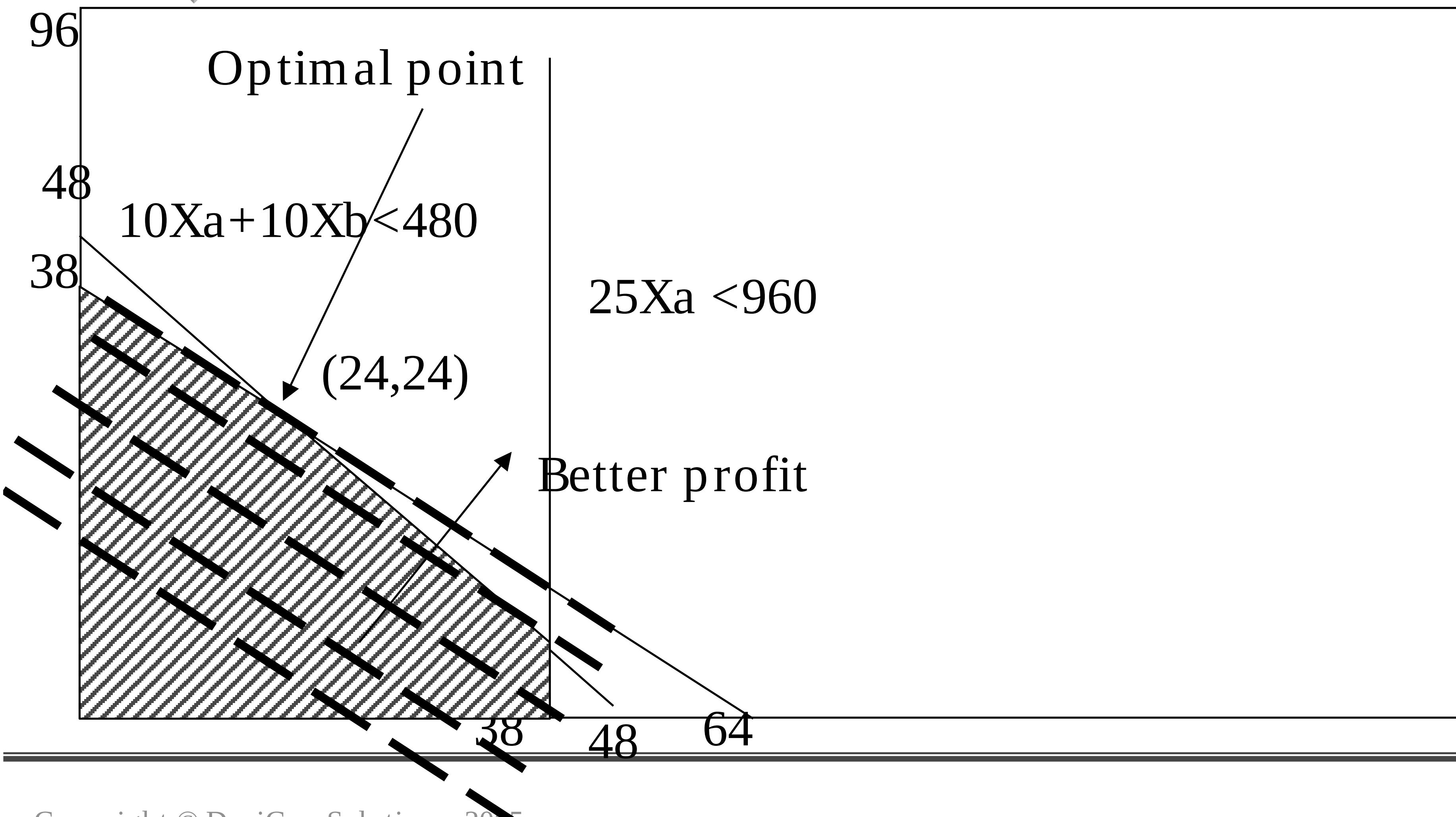
Product mix problem

- In a small shop there are one centering m/ c, four lathes and two milling machines
- The shop makes two product say A & B
- Time taken by each product is as follows:
  - Centering m/ c : 10 & 10 min
  - Lathe: 30 & 50 min
  - Milling: 25 & 0 min
- Profit on the Product is 10 & 15 respectively
- Find the best product mix

Understanding LP in 2D

- Let Xa & Xb the production quantity for product A and B
- Then profit: 10Xa+15Xb
- Constraint
  - Centering m/ c: 10Xa + 10 Xb < 480
  - Lathe: 30 Xa + 50 Xb < 1920
  - Milling 25 Xa < 960
  - Xa & Xb > 0

Graphical solution of the problem



Problem formulation in LP

- LP problems are stated as:
- Max/ Min : $c_1x_1+c_2x_2+c_3x_{3+}\ldots$
- Subjected to
- $a_{11}x_1+a_{12}x_2+\ldots < b_1$
- $a_{21}x_1+a_{22}x_2+\ldots < b_2$
- $x_1,x_2, \ldots > 0$

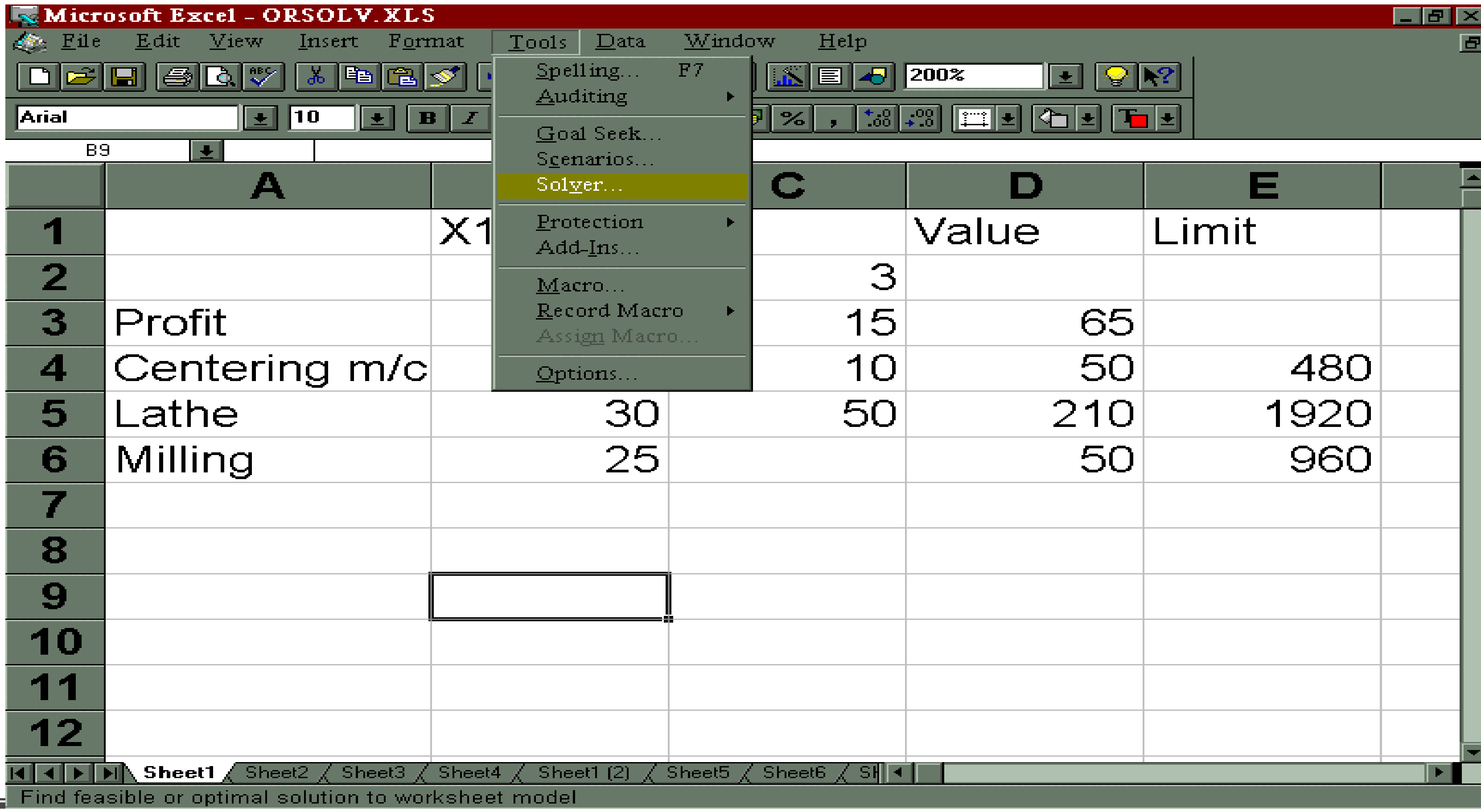
Available tools - Excel

- There are several mathematical procedure available to solve an LP. eg. Simplex, Revised Simplex, Big M etc.
- These procedure require large number of calculation, so we shall learn on solving LP on computer.
- For simple LP problems a rudimentory tool is available in "Excel". For bigger problem "Lindo" is the industry standard for LP problem solving

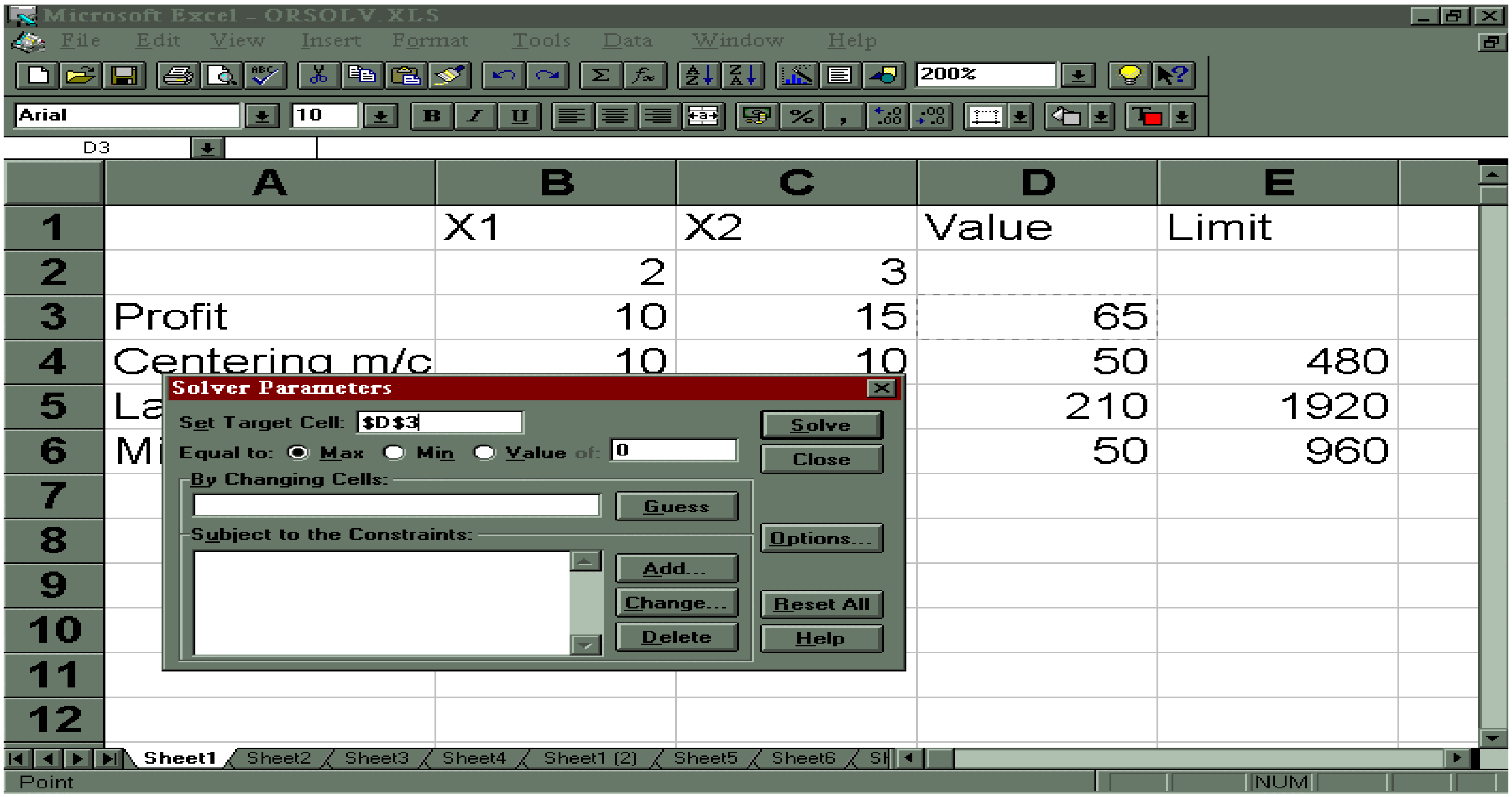
Problem definition

	A	B	C	D	E
1		X1	X2	Value	Limit
2		2	3		
3	Profit	10	15	65	
4	Centering m/c	10	10	50	480
5	Lathe	30	50	210	1920
6	Milling	25		50	960

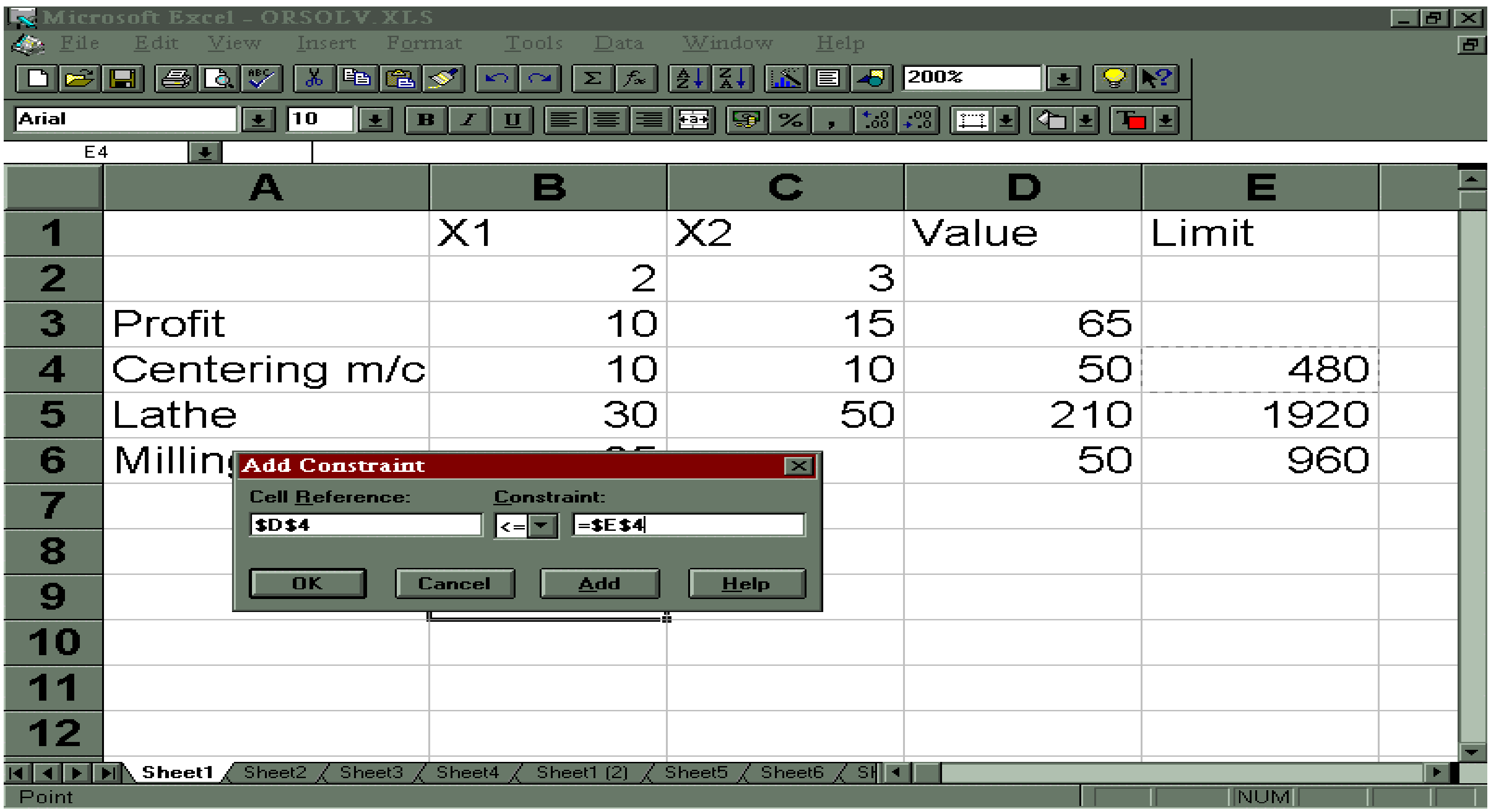
Call solver


Microsoft Excel - ORSOLV.XLS
File Edit View Insert Format Tools Data Window Help
Spelling... F7
Auditing
Goal Seek...
Scenarios...
Solver...
Protection
Add-Ins...
Macro...
Record Macro
Assign Macro...
Options...
B9
X1
Value
Limit
3
Profit
15
65
Centering m/c
10
50
480
Lathe
30
50
210
1920
Milling
25
50
960
Sheet1 Sheet2 Sheet3 Sheet4 Sheet1 (2) Sheet5 Sheet6
Find feasible or optimal solution to worksheet model

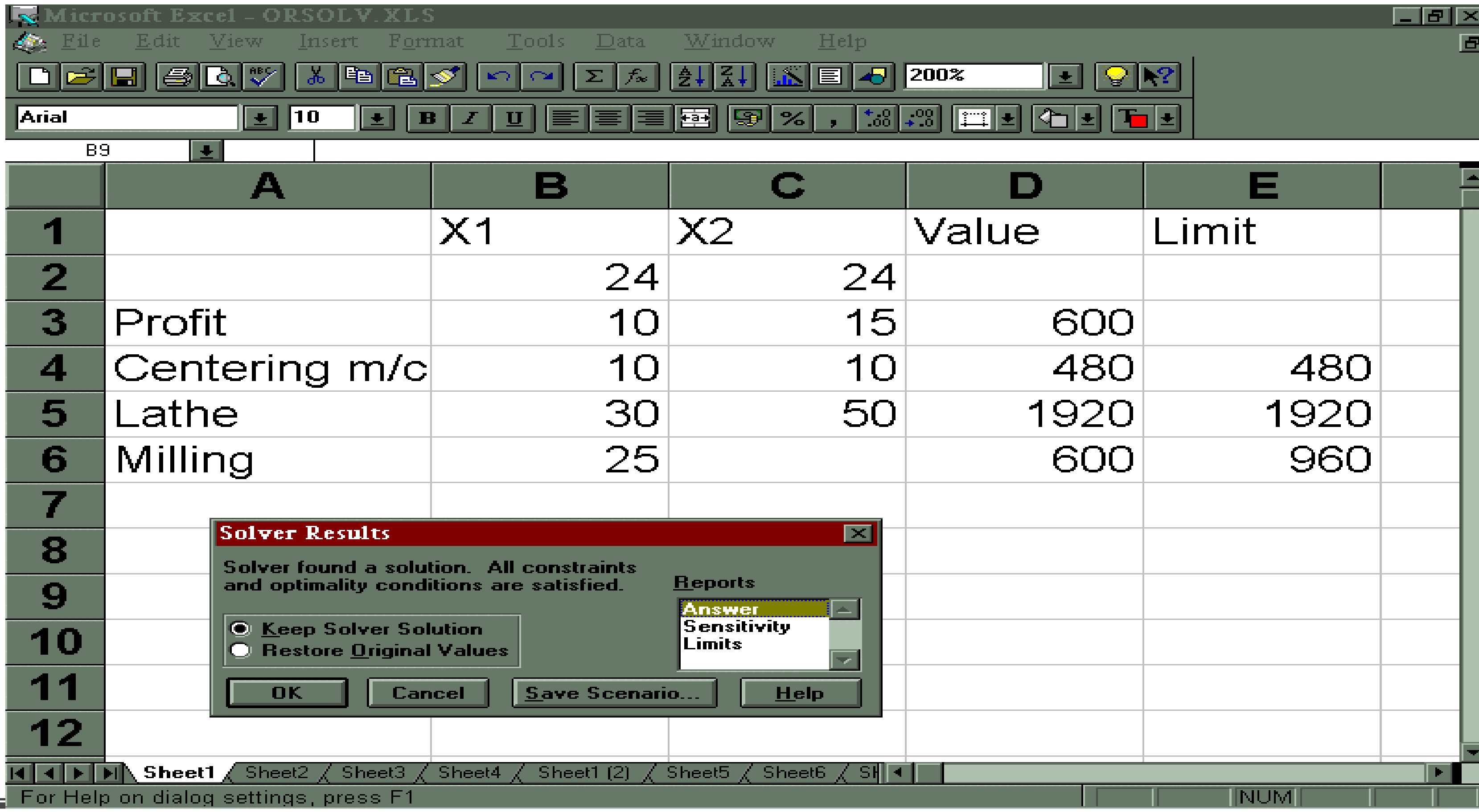
Define objective function cell



Define constraints



Answer of problem



	A	B	C	D	E
1		X1	X2	Value	Limit
2		24	24		
3	Profit	10	15	600	
4	Centering m/c	10	10	480	480
5	Lathe	30	50	1920	1920
6	Milling	25		600	960

Detailed answer

Microsoft Excel 5.0 Answer Report
Worksheet: [ORSOLV.XLS]Sheet1
Report Created: 9/5/99 18:32

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$3	Profit Value	65	599.9999895

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$2	X1	2	24.0000021
$C$2	X2	3	23.9999979

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$4	Centering m/c Value	480	$D$4<=$E$4	Binding	0
$D$5	Lathe Value	1919.999958	$D$5<=$E$5	Not Binding	4.19201E-05
$D$6	Milling Value	600.0000524	$D$6<=$E$6	Not Binding	359.9999476

Overview of other tools : Lindo

- Solver is a handy tool to solve small LP problem
- For problem with around 100 variable / constraints a better computational algorithm is required
- Industry standard package for LP is “Lindo”
- Data input and preparation is very similar to solver, but there are better diagonostic tools available to pin point any problem with data

Transportation problem

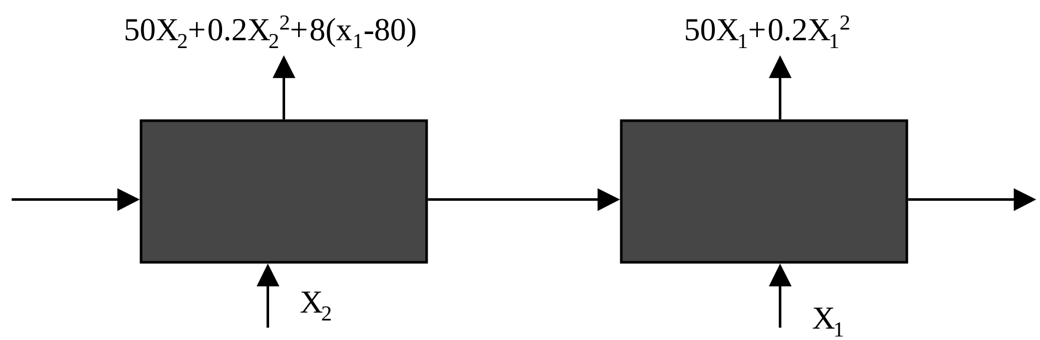
- It is a class of LP where some material is to be supplied from various plants to various markets
- General formulation of LP will make the problem very large even for 3 plants and 5 depots. This will mean 15 variables and 8 constraints.
- By using some underlying simplicity of the formulation these problems can be solved in very simple manner

Vogel's Approximation Method (VAM)

- Plot the unit cost elements in a from/ to matrix
- Find penalty for not using the cell with least cost for each row and column. ie. difference between smallest cost and the one next higher cost
- Allocate maximum quantity for the row/ col with maximum penalty
- Strike off the row/ col in which all supply/ demand is exhausted
- Recalculate penalty for the remaining
- repeat till all supply/ demand has been fulfilled

Dynamic programming

- Production requirement is given 80 for today and 120 for tomorrow. Suppose manufacturing cost is given by $50x+0.2x^2$ and inventory carrying cost is Rs. 8 per unit then determine best manufacturing strategy.



DP (contd...)

- This can be taken as two stage decision problem
- $R_1(x_1)= 50x_1+0.2x_12$
- Assuming X_1* gives inventory I_2 we get $X_1^*=80+I_2$
- or $R_1(X_1)= 50(80+I_2)+0.2(80+I_2)^2 = 5280+82I_2+0.2I_2^2$
- Cost in second month $R_2 = 50X_2+0.2X_2^2+I_2$
- Total cost: $R = 5280+82I_2+0.2I_2^2+50X_2+0.2X_2^{2\ +}8I_2$
- Now, $I_2 + X_2 = 120$
- $R = 18960 - 88X_2 + 0.4\ X_2^2$
- by $dR/\ dX_2 = 0,\ 0.8X_2^* - 88 = 0$
- $X_2^* = 110$ & $X_1^* = 90$

DP - Discrete problem

- Problem: Design most cost effective overhead water tank for 100 K Liter
- Parts of tank are:
  - Tank
  - Column
  - Foundation
- Here,
  - stage input is weight
  - Desicion input is our choice of alternative
  - Return fucntion is cost
  - Objective: minimize total cost

DP - Discrete contd ...

Stage 1			Stage 2						
Type	**Cost**	**Weight**							
Circ steel	15000	**105**	10T,8000 Steel	15T, 3000 RCC Spiral		30T,3000 RCC tied			
Circ steel w/d	12000	**110**			15T, 9000 Steel	30T,4000 RCC Spiral			
Rect RCC w/d	9000	**115**							
Rect steel	~~10000~~	~~**115**~~							
Rect RCC	**6000**	**125** →				20T,10000 Steel	**50T,6000 RCC Spiral**		
Circ RCC w/d	8000	**130**					50T, 5000 RCC tied		
Circ RCC	5000	**145**					30T,15000 Steel	60T,8000 RCC Spiral	70T, 6500 RCC tied
At Stage 2		Weight	**115T**	**120T**	~~**125T**~~	**145T**	**175T**	~~**205T**~~	**215T**
		Total cost	**23000**	**18000**	~~**21000**~~	**16000**	**12000**	~~**13000**~~	**11000**
Stage 3		Spread		20T, 500		25T, 2500	35T, 3000		60T, 5000
		Concrete		15T, 1000		20T, 1500	30T, 2500		55T, 3500
		Steel pile		5T, 1500		6T, 2000	**8T, 2000**		10T, 3000
Total cost				**18500**		**17500**	**14000**		**14500**

PERT & CPM

- These techniques are extensively used for project planning/ monitoring
- Terminology:
  - Critical path: The sequence of activities on which completion of the entire project is dependant
  - Slack: Time available in which delay in the job does not create any delay in project completion
  - Resource balancing: Scheduling activities to reduce resource requirement by making use of slack
- MS-project is a popular project monitoring software

Queueing theory

- Answers questions like
  - How many persons will be there in bank counter?
  - How much time the truck has to wait before entering through CRS gate?
  - What will happen to waiting time if we increase number of counters ?
- Let
  - a : arrival rate per unit of time
  - s : service rate per unit time when server is busy
- Then for single server queue
  - Number of unit in system = a/ (s-a)
  - Waiting time = 1/ (s-a)

Simulation

- Simulation is another very powerful tool for decision analysis
- Steps
  - Build computer model using relationship, statistical distribution of variables
  - Test model
  - Run model to gather statistics on effect of various decisions
- Various special purpose simulation languages available
  - GPSS is a good language for discontinious event simulation eg. shop simulation
  - Quest is another very powerful language for simulating shop layout, capacity etc
  - Dynamo is used to simulation economic models of continious variables
  - For quick and approximate analysis spreadsheet has many tools - what if table, goal seeking, random number generation etc

Clusturing

- Clustering is used to find relation in seemingly unrelated events
- Uses
  - layout of machines in flexible flow system
  - Identification of machine breackdown that occurs together
  - Identification of secret liason between different person

Where to get more information

- Books:
  - Optimization - theory and application by SS Rao, Wiley Eastern Ltd
  - Principles of Operations Research by Harvey M Wagner, Printice Hall
  - Liner Programming by HA Taha
  - System simulation, Narsingh Deo
- Journals:
  - Opsearch, Operatioal Research Society of India